The ELECTRE family (member of the Outranking methods)

The ELECTRE approach starts from the intuitively attractive premise that a Decision-Maker (DM) can only make approximate comparisons of the performances of alternatives. ELECTRE allows performances which are not numerically equal to be considered equal. Outranking does not have an axiomatic basis, but rather is based on parameters and a decision algorithm. It is still necessary for the DM to provide the analyst with scores for the alternatives against the criteria, but the preference system is “designed” via the approach. Thresholds are set which reflect the DM’s wishes to compare these performances in an imprecise manner. These are:

- The indifference threshold q_j
- The preference threshold p_j and
- The veto threshold v_j

The inclusion of q_j and p_j results in the introduction of zones of indifference and preference between preference scores. This accounts for any imprecision or uncertainty within the stated performances. The veto threshold indicates the point at which an alternative is so outperformed on one criterion that the hypothesis “α is at least as good as b” does not hold overall. The thresholds q_j and p_j can be constant for all levels of performances on criterion j, or can be proportional to the score $g_j(\alpha)$ of the alternative α on criterion j. The two forms can be combined to give $p_j = \mu\, g_j(\alpha) + \varepsilon$.

The indifference threshold, q_j, is a real positive number which represents the maximum difference that may occur between scores on each criterion to which the DM remains indifferent. This may expressed by the following condition:

$$\alpha\, \boldsymbol{I}\, b \Leftrightarrow \left| g_j(\alpha) - g_j(b) \right| \leq q_j \qquad (1)$$

where $g_j(\alpha)$ is the performance of action α on criterion j. q_j is likely to vary for each of the different j criteria.

The primary purpose of the introduction of the preference thresholds, p_j, is to recognize that the performance of one alternative is strictly preferred to that of another only if there is a sizable difference in their scores. Further, strict preference for one alternative over another on a specific criterion can only occur if the difference in evaluations on that criterion is sufficiently large to overcome any imprecision, uncertainty and errors of determination that may exist in the initial data. This may expressed as:

$\alpha \boldsymbol{P} b \Leftrightarrow g_j(\alpha) > g_j(b) + p_j$ (2)

Strict preference for action α relative to action b could be established based on just one criterion, irrespective of their relative performance on the rest of the criteria. This is the case if the deficit in the score of α compared to that of b on the particular criterion j is larger than the respective veto threshold v_j. We can write:

$b \boldsymbol{P} \alpha \Leftrightarrow g_j(\alpha) + v_j < b_j$ (3)

The performances of pairs of alternatives on the individual criteria can be examined. Using these thresholds, the performances can be categorized into one of a number of groups. The first is indifference, ***I***, and its members satisfy equation (1) above. ***P*** is the set where strict preference can be claimed and its members are given by equation (2). ***Q***, the set of weak preference, represents those alternatives that lie between indifference and strict preference. An alternative residual set, ***R***, is the set containing the alternatives deemed to be incomparable.

It is also possible to attribute an outranking relation, ***S***, to each of the criteria. This embodies the concept that one alternative can be considered to be "at least as good as" another:

$\alpha \boldsymbol{S}_j$ b: α is at least as good as b on criterion j $\Leftrightarrow g_j(\alpha) \leq g_j(b) - q_j$

$\{\alpha \boldsymbol{S_j} b\} = \{\alpha \boldsymbol{P}_j b\} \cup \{\alpha \boldsymbol{Q}_j b\} \cup \{\alpha \boldsymbol{I_j} b\}$

Using these groupings, specifically ***P***, ***Q*** and ***S***, ELECTRE specifies values known as concordance and discordance coefficients. In general, the concordance coefficient is a measure of the strength of the arguments that validate the concept "α is at least as good as b" taking into account all the evaluation criteria. If concordance measures the strength of support for the hypothesis *α is preferred to b*, then the discordance coefficient measures the strength of evidence against this hypothesis.

Unlike the weights used in Multi-Attribute Utility Theory - MAUT (see Deliverable 1 for further details), which represent the relative importance of the criteria, the weights specified in ELECTRE do not express tradeoffs. They are ordinal only and are a measure of whether a criterion is of greater, equal or lesser importance than another. Each criterion weight is designed to be constant for all possible scores on that criterion. These weights are incorporated into the analysis via the expressions for concordance and discordance.

ELECTRE allows for the alternatives to be compared indicating whether an "optimum" exists, whether certain alternatives can be discarded, or whether further investigation should be undertaken. If an "optimum" exists, then a particular alternative will outrank all the others. If all the others outrank an alternative, then it can be discarded.

Outranking methods can be utilized to give results in one of the following perspectives: selection of the *best action* according to input; assignment of each alternative to categories as "accepted", "rejected", "needs further investigation"; ranking of the most satisfactory alternatives as defined by some pr-ordering. There are five ELECTRE models which can be used depending on which of the above perspectives the DM is willing to adopt.

The ELECTRE III method

The ELECTRE III method starts by comparing each action α with each other action b, with the aim either to accept, reject, or more generally to access the credibility of the assertion: *action α outranks action* b, or α ***S*** b.

The calculation of the credibility S(α, b) of this affirmation is based on common sense: the formula determining the value of S(α, b), over the interval [0,1] is constructed in such a way as to respect certain qualitative principles, and in particular excludes the possibility that a major disadvantage on one criterion might be cancelled out by a number of minor advantages on other criteria. The formula is essentially based on the concepts of concordance and discordance, as described earlier. The goal is to use the values of q_j and p_j in order to respectively:

- characterize a group of criteria considered to be in concordance with the affirmation being studied and access the relative importance of this group of criteria compared with the remaining ones, and
- characterize, amongst the criteria that are not in concordance with the affirmation studied, the ones whose opposition is strong enough to reduce the credibility which would result from taking into account just the concordance, and to calculate the possible reduction in it that would thereby result.

To be able to carry out such calculations, one must first express in explicitly numerical form:

- The relative importance k_j that the DM wishes to confer to criterion j in the calculation of discordance. The only effect of these indices of importance is the order they induce (because of their being added together) on the groups of criteria affecting the calculation of concordance.

- The minimum value of discordance, which gives criterion j the power to take all credibility away from the affirmation being studied, even when opposed to all the remaining criteria in concordance with the affirmation, is called the veto threshold of criterion j. It is not necessarily a constant and will accordingly be denoted $v_j[g_j(\alpha)]$.

In ELECTRE III, the indices of importance are not part of a system of total compensation. Their purpose is still to represent the degree of importance the DMs wish to accord the different criteria, but two distinctive principles are at a play:

This wish is generally expressed in terms of purely qualitative judgments of preference that are unsuited for allocating a precise cardinal value to each of the indices of importance (which are therefore only endowed with an ordinal value).

These indices of importance affect only the concordance, and the information about them can be obtained by studying only situations that limit the discordances to such an extent that the veto phenomena do not come into account.

These coefficients are calculated as follows:

Concordance index:

$$c_j(a,b) = \begin{cases} 1 & g_j(b) - g_j(a) \le q_j \\ 0 & g_j(b) - g_j(a) \ge p_j \\ \dfrac{p_j + g_j(a) - g_j(b)}{p_j - q_j} & q_j \le g_j(b) - g_j(a) \le p_j \end{cases}$$

and discordance index:

$$d_j(a,b) = \begin{cases} 0 & g_j(b) - g_j(a) \le p_j \\ 1 & g_j(b) - g_j(a) \ge v_j \\ \dfrac{g_j(b) - g_j(a) - p_j}{v_j - p_j} & p_j \le g_j(b) - g_j(a) \le v_j \end{cases}$$

And introducing the weights, we derive from the concordance indices:

$$C(a,b) = \frac{1}{\sum_{j=1}^{r} k_j} \sum_{j=1}^{r} k_j c_j(a,b)$$

It should be stressed that both the indices of importance and the veto threshold are not numbers derived from numbers that exist in reality. They are merely designed to represent the DM's deliberate policy decisions, which are necessary of a qualitative nature. Because of this there is a non-negligible degree of arbitrariness in the values chosen. In other words, decision aid must take into account whether the results of the calculations remain robust when this element of arbitrariness is allowed to vary.

Given any two actions, the numerical calculation of S(α, b) defines what is called a fuzzy outranking relation. This relation summarizes the results of the comparisons of all possible pairs of actions. These results are established from the data available using the qualitative principals (notably of non-complete compensation) that go to make up he credibility of the outranking S(α, b) near 0 or 1 represent a solidly established relation, whereas S(α, b) near ½ indicate an easily-upset affirmation.

The relation is as follows:

$$S(a,b) = \begin{cases} C(a,b) & d_j(a,b) \le C(a,b) \\ & \forall j \\ & d_j(a,b) \ge C(a,b) \\ C(a,b) \cdot \prod_{j \in J(a,b)} \dfrac{1 - d_j(a,b)}{1 - C(a,b)} & J(a,b) : d_j(a,b) > C(a,b) \end{cases}$$

ELECTRE III consists of taking these results and using them to construct a partial preorder, one, in other words, that doesn't necessarily have to be as complete as it goes when the method of aggregation consists of producing a single criterion. Given any two alternatives α and b, a complete preorder ranks α:

- either before b,
- or equal to b
- or after b

The partial preorder adds a fourth possibility: α is not situated with relation to b. This represents the fact, given the values of S(α, b) and S(b, α) and of S(α, c), S(c, α), S(b,

c) and S(c, b), with $c \neq \alpha$ and $c \neq b$, good reasons may be produced for ranking α after b.

The fact that in some cases it is not possible to come to conclusion about the relative position is no a weakness of the method. It is the natural consequence of not only the imperfect nature of the data (which must not express more information than they contain) but also of the precautions taken in aggregating the criteria (precautions taken with the purpose of avoiding expressing unjustified global preferences).

The general approach for the exploitation of the valued outranking relation is done by constructing two preorders Z_1 and Z_2 using a descending and ascending distillation process (respectively) and then try to combine these to produce a partial preorder $Z = Z_1 \cap Z_2$. The descending distillation is as follows:

Let $\lambda = \max_{a,b \in A} S(a,b)$. Determine a "credibility value" such that only values of S(α ,b) that are sufficiently close to λ are considered; Thus if λ = 1, let s(λ) =0.15. Define the matrix T as:

$$T(\alpha, b) = \begin{cases} 1, & if S(a,b) > \lambda - s(\lambda) \\ 0, & otherwise \end{cases}$$

Further, define the qualification of each project –Q(α)- as the number of projects that are outranked by project α minus the projects which outrank α. Q(α) is simply the row minus the column sum of matrix T. The set of options having the largest qualification is the first distillate of D_1. If D_1 contains only one element, repeat the previous procedure with $A \setminus D_1$. Otherwise, apply the same process inside D_1. If distillate D_2 contains only one alternative, the process is started in $D_1 \setminus D_2$ (unless it is empty); otherwise it is applied within D_2 and so on until D_1 is used up. The procedure is then repeated starting with $A \setminus D_1$. The outcome is the first preorder Z_1; the descending distillation. The ascending distillation is carried out in a similar fashion expect that the projects with the smallest (rather than the largest) qualification are retained first.