Abstract

Education as it tends to be offered today at international level has not accomplished to formulate with explicit scientific way two of its most fundamental issues one being the correct use of knowledge and the other being the scientific derivation of boundaries of wrong and right. This work searches and defines the boundaries of wrong and right so that they have a wider acceptance and a diachronic value. The work is based on scientific methodology using mathematics to model human error and is trying to maintain all developments within human dimensions. For this reason two fundamental structures developed by the Plato and Aristotle are used and are further developed employing modern scientific bases incorporated within a mathematical model. These two structures provide definitions and ideals to be used as foundations to build education where knowledge will be used correctly to maintain peace, to improve the quality in the environment and therefore to add quality in life. The results of this work lead to basic principles of modern education for a peaceful world with quality of life and consequently a virtuous world.

Keywords: education, philosophy, mathematics, virtue, human error, wrong/right boundaries, human dimensions

Introduction

The educator E. Papanoutsos, 1984: 14-16, in the introduction of his Book Practical Philosophy reports the following: The reader or the listener of courses in philosophy when closing the book or leaving the classroom has the feeling of an empty space. Consequently, Papanoutsos, 1984: 14-16, is referred to the ancient Greeks saying that … they never became theorists of the study-room as do modern philosophers. Actually, the ancient Greeks developed human centered bases of philosophy and maintained such material within human dimensions so that their philosophical approaches did not leave empty spaces. However, education in our days, because of this empty space, has a severe deficiency on philosophical bases and has failed in educating people to use their mind energy in the right direction with a result such energy either is nullified when half of it is wrong and the other half is right, or in other cases, it is negative (corruption, social injustice, wars, etc.). A current example of mind energy nullification is when a group of people is educated to destroy another group of people and reversely.

However, because this subject is quite lengthy here we will deal with the hypothesis: Whether it is philosophically feasible to define the boundaries of wrong and right so that education has a diversity options in providing alternatives for the correct use of mind energy, as is the knowledge, thus maximizing its constructive use and minimising its destructive use.
If, for example, it is proven that at this moment the natural environment of planet earth is deteriorating because of human activities, then the energy of human mind is evaluated as more destructive than constructive.

This present work makes an effort to search for the elements of this hypothesis, to give answers and solutions with diachronic value and wider acceptance, working within human dimensions. This research is based on specific work of ancient Greek writers and it rises to prominence two philosophical structures: one of Plato (from the Republic), and one of Aristotle (from the Nikomachean ethics) which can be considered as the foundations for education.

This work reveals also that mathematics is the most important tool of human mind for the scientific analysis of any structure so that such analysis and study is sufficient and complete. Mathematics can also work to increase mind’s value in its effort to do scientific research by holding mind’s power within human dimensions and by avoiding wasteful thinking.

**Education ideals**

As mentioned in the introduction, ideals are provided by two philosophical structures which are chosen as foundation for education and may also be considered to have similar or higher value for human sciences as Newton’s Law of gravity and Einstein’s theory of relativity in physics. Both of these structures have diachronic validity and global acceptance and will be analyzed using mathematics.

Plato in his book “The Republic” defines education as: "the therapy of the spirit ... and as, when the body is sick, it needs medical treatment, when the spirit is sick, it needs education". Consequently, Human spirit structure is defined by Plato as: "...human spirit consists of three components or three states that analytically are: logic, desire, and anger". Plato completes the definition of education by defining healthy spirit as follows: "... logic keeps control over and balance between desire and anger". To clarify the meanings of control and balance, Plato gives the following example: Compares spirit with a car pulled up by two horses - a blind horse representing desire and a crazy horse representing anger - and the coachman - being the logic - who keeps control over and maintains a balance between these two horses in order to move the car to the correct way (which is the way of virtue, Hatzopoulos J. N., 2004: 19).

This example about healthy human spirit as a mathematical structure (see Figure 1b) can be expressed by a rectangular triangle where the hypotenuse is logic and the two other sides represent desire and anger. A mathematical relation of healthy spirit can be established by Pythagoras theorem. This structure of human spirit resembles also a three dimensional coordinate system where the three coordinates (X, Y, Z – see Figure 1a) can express the position of all points in the three dimensional space. Similarly three components (R, G, B – see Figure 1a) of primary colors are needed to express all color hues. As shown in Figure 1b, the same thing could happen with Plato’s three components of human spirit where all states of human mind M_S (feelings, joy, happiness, sorrow, imagination, etc.) can be expressed by three components (D, A, L – see also Figure 1c).

Plato’s structure about education could be considered as the effort to maintain an internal balance so that logic L_B is balancing desire and anger according to Pythagoras theorem (see Figures 1b, 1c) as follows:

\[ L_B^2 = D^2 + A^2 \]

This equation indicates the ideal state of human mind where no error is committed.
However, the actual or current state of mind $M_S$ is expressed by its three components $M_S(D, A, L)$. In this way human error can be defined as the difference between current logic and balancing logic (see Figure 1c) as follows:

$$\text{Human error} = L - L_B$$

![Figure 1](image_url)

Figure 1. A mathematical analysis of the structure of education as an effort to balance the states (components) of human mind.

As indicated by this equation, human error could have either a positive sign or a negative sign depending on the values of $L$ and $L_B$. It is also important to understand that if human error is within certain limits then it would be considered as “correct”. These limits define the boundaries of wrong and right and they will be determined bellow.

It is recommended that researchers in human sciences must try to quantize and evaluate most common states of human mind based on these quantities ($L$, $L_B$, $D$, $A$) and using existing data from human sciences such as education, history, sociology, political sciences, law and justice, etc.

Modern definition of education is mostly dealing with the production and transfer of knowledge without specific ideals based on philosophical foundations and because of this it creates a controversy and does not help a person to focus and direct most actions in the correct direction (i.e., sustainable development). There are many cases today where groups of people are trained to destroy other groups of people and reversely, which means that there is no clear definition of education because there is no clear definition on what is wrong and what is right and the wrong / right relation to human error. Furthermore, the current destruction of the environment which results to rapid climate changes indicates that there is no sufficient education to the people and consequently to the decision makers which act the wrong way because there is no sufficient public reaction. However, the only way to protect and save the earth’s environment is to educate the public in an effort to minimize human error.

As stated above in Plato’s example the correct way to move the car was the way of virtue (Hatzopoulos J. N., 2004:18-19). However, virtue is defined in the following paragraph.
Virtue ideally represents the correct human action and as a structure is defined and analyzed in all its extent and in all its depth by Aristotle in his work *The Nikomachean Ethics*. According to Aristotle, Virtue is: *mesotita* (a midway) and is to be found in midway, in between two extreme actions or "badness". Aristotle then gives the following example in order to clarify the structure of virtue: If bravery is a virtue then the brave person is to be found in midway, between the provocative and the coward person, *and when one is brave, then the coward will call her/him provocative because she/he is beyond coward’s capacity, while the provocative will call her/him coward because she/he is beneath provocative’s capacity*. Accordingly, one could characterize thrifty as a virtue that is to be found in midway between stinginess and overspending and the stingy will call the thrift as overspender while the overspender will call the thrift as stingy.

Aristotle also defines the person of virtue as *the one who is trying to be a person of virtue* which means that virtue is the effort to maintain actions within the midway and which allows anybody at any moment to be a person of virtue (*never is late*). According to Aristotle the person of virtue is not the one who commits no errors but is the one who is trying to minimize human errors. This definition of virtue is completely fitted within human dimensions and under certain conditions it may allow extreme actions to take place, as is for example, self defence.

**Mathematical analysis of boundaries of wrong and right**

Examining more carefully the example given by Aristotle about the brave person, who is a person of virtue, then she/he will be considered by the coward as provocative, which means that the coward, believing that she/he is a person of virtue, underestimates virtue and therefore mathematically is committing an error with a negative sign. On the contrary, the provocative considers the person of virtue as being a coward and accordingly overestimates virtue hence she/he mathematically is committing an error with positive sign. It is evident that humans, by their own nature make errors because human brain is anatomically based on neurons and such systems are not absolutely correct. For example (see Figure 2), if one walks over a flat road and meets a small obstacle like a rock having the size of a football, then the way one rises the foot to pass over the obstacle is different each time one walks over. There is an optimum way to pass over the obstacle with minimum energy (zero error) which can be approximated with practice but it will never be followed exactly. There is a lower limit and an upper limit (see Figure 2) to rise the foot to pass over the obstacle with optimum energy (correct, virtue) without having a false step. Finally there are many cases outside these limits where it takes place a false step (negative and positive error). How bad the false step is depends on the damage caused to this person (absolute value of error). It must

![Figure 2. The boundaries of human error and the correct as a midway of virtue.](image)
be noted that, repeating an effort, neurons are trained and constantly improve their performance but they never become perfect. This example may help to define precisely the boundaries of wrong and right where wrong occurs by a false step and right occurs with the effort for optimum energy.

The Aristotelian midway of virtue has a universal validity, for example, taking into consideration the orbit of the earth around the sun, one may observe that the earth will never follow exactly the same path and there is a midway where orbits of the earth must occur in order to have equilibrium. If the earth gets off such bounds towards the inside, then the earth may collide with the sun, if the earth gets off such bounds towards the outside, then the earth may get lost in space. This example defines also precisely the boundaries of wrong and right where wrong occurs when the earth tends to collide with the sun (negative error) or tends to get lost in space (positive error) while right occurs within the midway of orbits which follows until now.

Having this analysis in mind, then human error structure can be expressed mathematically as follows (Hatzopoulos, 2004: 54): We may establish an axis X (See Figure 3), consisting of three straight line segments:
(a) The segment on the left called “Error on the left (ML)”, which measures the degree of error or badness - human error and shows the amount of underestimation of virtue (negative error);
(b) The intermediate segment in the middle called “Right Logic (RL)” or virtue (correct).
(c) The segment on the right called “Error on the right (MR)”, measures the degree of error or badness - human error and shows the amount of overestimation of virtue (positive error).
(d) The location Xo with zero error is called location of Supreme Being, which is a mathematical entity, because no human being is considered as having zero error.

As shown in Figure 3, the distribution of human error is symmetric with respect to the central point Xo of zero error in the intermediate segment – midway of virtue. However, human error is expanded from point Xl to minus infinity and from point Xr to plus infinity. If humans would be able to estimate precisely their error they would choose a specific location along the X-axis. Notice that many times people estimate such a location as the political parties, social groups, community clubs and their followers. Court decisions also estimate such locations as a result of the function of the justice system. It must also be noted that the estimation of location of boundaries of midway of virtue has to be subjective in order to stay within human dimensions and therefore requires a wider consensus (stochastic model) because each individual may have a different opinion on the subject and such a consensus may be valid only if there is a minimum bias in expressing such opinion. Minimum bias may be considered if the voters have an education whose effort is to develop a healthy mind as defined by Plato. Such matters about the influence of bias are discussed in detail by (Hatzopoulos, 2004: 70). Consensus means democratic procedures and voting which are philosophically founded as an effort to define the midway of virtue.
It is important to understand that since any human action includes an error \( X \) it means that this action to a certain degree is correct \( Y \) so that wrong \( X \) and correct \( Y \) coexist within such action. If wrong and right are to be quantized and related to each other, then they must be inverse proportional quantities to each other. A simple function to express this is as follows:

\[
Y = \frac{1}{X}
\]  

(1)

Where \( X \) is the wrong or error and \( Y \) is the correct or right.

From Equation (1) it is evident that:

\[
\text{for } |X| \to 0 \text{ then } Y \to \text{infinity.} \quad (2)
\]

Since \( X_L, X_R \) are located on the common boundary between wrong and right, then both variables \( X \) and \( Y \) must have exactly the same value (Hatzopoulos, 2006: 328) on this common boundary. However, we look for a value in X-axis where:

For boundary \( X_R : X = Y \) and for boundary \( X_L : -X = -Y \)  

(3)

Applying the boundary condition (3) on Equation (1) we have:

\[
X = \frac{1}{X} \text{ or, } X^2 = 1 \text{ or, } X = \pm 1
\]  

(4)

This determines precisely the borders of midway of virtue (Hatzopoulos, 2004: 59) as having values:

\[
X_L = -1, \text{ and } X_R = +1
\]  

(5)

Considering that such borders are defined by voting and the probability density function of human error is \( f(x) \) then as a stochastic model may be chosen the Gaussian standard normal distribution \( (\mu = 0, \sigma = \pm 1 \) Hatzopoulos, 2004: 63).

**Supreme Being**

As indicated by Relation (2), the mathematical entity named *Supreme Being* has a virtue with magnitude approaching at infinity and, consequently, it is not possible for this Being to have even a minimal badness at all times. Consequently, if we accept that Supreme Being has any
of human weaknesses at any time, we immediately depart from the location X=0 and thus we have not just one but numerous such beings like human beings.

Putting on the same diagram shown in Figure 4 both the error function X and the correct (virtue) function $Y = 1/X$ (Y-axis is perpendicular to X-axis) one may notice that if X takes values from -1 towards zero, then Y moves towards minus infinity. On the other hand, if X takes values from +1 towards zero, then Y moves towards plus infinity. This indicates that the Supreme Being is found in one single location of the X-axis and has a virtue which covers all values in universe from minus infinity to plus infinity. This is one and unique location because if departing even with a small amount from location zero, say $0 + e$, or, $0 - e$, where e is a very small positive real number, then there are many beings in such location with human weaknesses and not a supreme being.

**Looking for a peaceful world**

According to the analysis performed, if all humans were asked to vote for the location of midway of virtue then there are two possible cases to examine. The one case will represent the ideal society where the votes will follow the Gaussian standard normal distribution and the second case will represent the real society as it looks like today and will follow the form shown in Figure 5 (see also, Hatzopoulos, 2004: 73).

![Figure 5. The real society composed of various groups with corresponding biases $\mu_1, \mu_2, \ldots, \mu_n$.](image)

In Figure 5 the X-axis represents the human error and the Z-axis, which is perpendicular to the X-axis, represents the number of votes at a particular location on the X-axis.

The degree of error or bias in every social group is expressed by the mean value $\mu$ of the voters in this group. Moreover, the mean value $\mu$ expresses the size of the lever that creates the dynamics or the energy of each individual group. The energy (E) can be expressed as the product of the area ($A$) under the distribution curve of the group times the lever ($\mu$) of the group:

$$E_i = A_i \mu_i$$

In this case, the ideal society has a mean value $\mu = 0$ and zero bias. When a social group creates a bias in the interval -1 \ldots + 1, which is also the interval of virtue or correct, then its dynamics to claim rights at the expense of other social groups is practically zero because the lever magnitude is less than one. As a result, these social groups must have a large percentage of active citizens (voters) in order to survive. On the other hand a relatively small social group with a large bias $\mu = \text{Large value}$, could achieve a tremendous destructive power.

The state of peace is defined by the status of symmetry in Figure 5. If there is symmetry, there is also peace. Bias causes the departure of function f(X) from its symmetric structure. Any departure from symmetry creates a potential state of conflict in the population. It must be noted that extreme ideas gaining ground in certain groups of people tend to create asymmetry.
in the structure of X and Z axes. Asymmetry causes social conflicts and collisions which may lead to revolutions, wars and terrorist activities. On the other hand, symmetry creates peace which is more stable, if the distribution function of voters tends to follow the standard normal distribution where about 70% of voters are people of virtue, or,

\[-1 \leq X \leq +1 \text{ and } X \neq 0.\]

Symmetry may also create an unstable peace if voters outside the virtue limits are much more than 30%. In this case, if an extreme group with positive error appears, then an extreme group with negative error also appears to balance such action thus tending to create an unstable peace.

**Didactics**

According to the previous analysis mathematics were used to describe philosophical structures of Plato and Aristotle about education. Therefore, mathematics is a valuable tool of human mind to perform analysis and synthesis of simple or complicated structures (James Franklin, 1995). Consequently, it must be realized that taking out mathematics from any course in education the scientific analysis to be performed could be incomplete. This is very important for those who want to improve didactics. Looking at student’s status one may see that about a 20-30% are talented students in mathematics and they understand and learn theories right away and therefore for those students there is no problem. The majority of students 70-80% need more help to understand mathematics. A didactics method, which could help such students, is to use application examples from every day’s life (Hoyles, C. and Noss, R. 2003: 323-349, Noss, R. and Hoyle, C. 2007: 15 – 25, Hoyle, C. and Noss, R. 2007: 27 - 43) and such a method is known as the R.E. Gross problem solving method (Gross R. E., Zeleny L. D., 1958: 341-367). A complete example of this method is given by Manolas E., 2006: 292-296. R.E. Gross method if combined with computer programming to obtain quick results on complicated application problems could improve didactics and could also help students to advance into the new technology era being smarter than the machine and avoiding being button pushers.

Working on students at University level one may be the recipient of all problems students have from elementary school to the high school and senior high school in mathematics. Trying to decode the problems students have in this area it is evident that such problems are attributed to wrong attitude, wrong didactics and wrong practice followed by most educators in mathematics as follows:

1. They do not realize that mathematics is the science of structures (James Franklin, 1995) and as such is a tool of human mind.
2. Because of attitude #1, they do not orient their didactics towards applications to create motives to the students but instead they spend their time on theories which for the talented students there is no problem but for most of the students it does not make sense and they miss the substance creating gaps which generate opposite feelings and hateness about mathematics.
3. Many mathematicians particularly the good ones do not have experience about applications in mathematics and they usually do not like applications.
4. Computer programming in a simple computer language, such as Visual Basic for example, is not present in curricula of elementary and secondary education schools.

Pedagogy as a science must be able to adapt itself at least in cosmogonic changes when they take place. Since the decade of 1980 and afterwards a cosmogonic change took place which brought in the foreground an amazing human invention tool and this is the personal computer. The fact that this tool was evolved to a personal use from a team of young people
that manufactured the Apple computer (Leigh Kimmel, 1998), shows how big can be an offer to humanity by an insignificant team of people when they know how to use their free mind. IBM Company which was a colossus in computers at that time was waiting to evaluate the development of Apple computer in order to make its own move. Despite the IBM PC movement and the Microsoft which developed the IBM PC operating system, the team of young people that made the Apple system got ahead to accomplish the manufacture of Apple Macintosh whose characteristics Microsoft managed to approach ten years later and never up today was able to exceed it.

The right use of information technology presupposes good knowledge of computer and its capacities and over all the ability to develop software. The correct pedagogic process could state that: whoever is educated to use the computer as a tool is always more intelligent than the computer. This objective could drop the myth about computers and help students to develop the power of knowledge to use the computer as an effective tool in all scientific fields (natural sciences and human sciences) and in all kinds of activities. The largest advantage of computer as a pedagogic tool is that in any subject or problem or work can give fast results and in this way creates motives for the students to study with higher attention the scientific bases. If students have the ability to develop software, then they can study each piece of scientific work or research by testing it in the computer and in this way they obtain an essential confidence that they know this scientific part of the study or research. Young people must understand the example of the team which developed the Apple computer and they must realize that at any moment they can move ahead and take initiatives using their free mind putting objectives and standards thus helping themselves and the society. However, a very good knowledge in mathematics and computer programming (object oriented programming) could provide human mind with the best tool to perform analysis and synthesis of structures in any scientific field (natural sciences and human sciences).

If those issues are taken seriously into consideration then it is anticipated to have improvement to the current situation. An ideal improvement could be to obtain a percentage of 60-70% of students to understand and learn mathematics and computer programming. It must be noted that probability between $\sigma = -1$ and $\sigma = +1$ in standard normal distribution is within this percentage range.

Historically, mathematics had an essential role in education up to 520 A. D. where there was no segregation between human and natural sciences and science was studied as a total with philosophy physics, mathematics and human sciences being an integral set of human education. By 520 A. D. the Roman emperor Justinian closed the philosophical schools of Athens which were known to have in their main entrance the sign:

ΜΗΔΕΙΣ ΑΓΕΩΜΕΤΡΗΤΟΣ ΕΙΣΗΤΩ
to enter for those they do not know geometry).

Since then the dark ages come which lasted roughly for about 1000 years. Afterwards the needs in big discoveries and explorations brought again in the limelight the use of mathematics in relevant applications concerning natural sciences while human sciences were separated and were studied without or with minimal use of mathematics. However, up to today most human sciences do not use as much as it suppose to be mathematics resulting to an insufficient study in these fields. Consequently, the threatened destruction of the environment of planet earth from anthropogenic activities requires more activation of human mind and the use of mathematics in human sciences can contribute considerably in this direction.
Conclusions

This work dared and achieved within human dimensions an important goal for education which is the effort to develop a healthy mind with minimised bias, which is capable to understand and locate precisely the boundaries between wrong and right. Moreover, the Aristotelian midway of virtue and the effort of education to create virtuous persons were proven to contribute to the peace and quality in life by maximizing mind’s constructive energy and by minimizing mind’s destructive energy. Based on such analysis the correct use of knowledge can be achieved keeping in mind the following basic principles which are proposed to be adopted by all schools at global level for the education of all people:

1. **Education** is the effort to develop a healthy thought (mind) to the virtuous person.
2. **Training** is the effort to develop a healthy thought (mind) to the virtuous person in a specific subject, for example, the environment.
3. **Thought** is a composite of three states of human mind before it develops any action. These states are: **logic**, **desire** and **anger**. Thought determines all actions of a person and precedes to these actions. Any state of mind (joy, happiness, sorrow, imagination, etc.) is described by these three components.
4. **Healthy thought** is developed when there is an effort so that the logic state of a person’s mind balances the two other states which are the desire and the anger. If logic state does not balance desire and anger then there is a bias or human error. Balancing takes place within tolerance limits as defined by midway of virtue (within the boundaries of wrong / right).
5. **Healthy action** is the effort so that such action is shaped by a healthy thought and is virtuous.
6. **Virtue** is the effort so that the action of a person follows a midway path, which is found in midway between two extreme positions or badness. For example, thrifty is a virtue and is found in midway between stinginess and overspending. Virtue can be considered as "the effort to maximise the use of mind energy for constructive aims and minimise its use for destructive aims". Some times though, destructive actions are inevitable as for self defence.
7. **Virtuous person** is the one who tries to be virtuous, that is to say, the effort to maintain the midway of virtue. This definition is within human dimensions and allows to anybody at anytime (**never is late**) to be virtuous.
8. **Virtuous action** is the effort so that corresponding action follows the midway of virtue.
9. **Justice** is the top virtue and includes all virtues.
10. **Democracy** is the process to define the midway of virtue. Consequently, for defining the midway of virtue it is required a wider consensus of educated people with minimum bias or error.

These foundations are capable to support the building of education and they must be adopted by all schools at global level so that they become the property of all people. These foundations can support the reformation of educational programs at all levels and also the education of teachers. In this way a person will know the right directions to spend the mind energy maximizing the degrees of freedom by having a great diversity of correct choices. The quality in life can be upgraded and a person will know how to be an active citizen and have more participation in the challenges at the globalisation era. A person will also know how to make a better sustainable management of resources in order to be able to support groups of people who need more help.

The analysis for the Supreme Being shows the existence of absolute harmony and perfection in universe something that people try to locate without success and attempting to
do so they get lost because perfection is out of human dimensions. However, using mathematics such absolute harmony and perfection is located in a completely ideal form.

Mathematics should be considered officially at a global scale as: a tool of human mind to analyze and synthesize simple or complex structures. Mathematical relations connect the elements of such structures and help to estimate values of elements which otherwise are unknown and become known from their relation to other known elements (they are functions of other known elements). Didactics should achieve an objective where 60% - 70% of students at all education levels must be well educated in mathematics. Mathematics should also be used to analyze structures in human science fields in order to advance such fields with scientific sufficiency.

A well developed knowledge in mathematics and computer programming (object oriented programming) could provide human mind with the best tools to perform analysis and synthesis of structures in any scientific field (natural sciences and human sciences). Mathematics could also help human mind to avoid wasteful thinking thus minimizing stress, depression and most of modern psychological problems.

Looking at things within human dimensions one could realise that the highest power in universe is human mind and perhaps the entire universe could have no significance without the human mind. Consequently, it is imperative that education must make the effort to maximize the creative ability of human mind particularly in the area of scientific research that is to say in the area of truth and to free mind from all kinds of bonds which are out of human dimensions such as fantasies and lies which create traps like blackmails, phobias, erroneous information and false promises. At this point mathematics as a tool can provide valuable help to free human mind as, for example, is shown in Figure 4 where is presented a mathematical expression for the supreme being and is proved that it represents the perfection and harmony without any human weakness.

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