Application of mathematics to analyze classic science structures and impacts on didactics and education.

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ABSTRACT

This paper deals with the use of mathematics as a tool of human mind to analyze structures. This idea is found to improve the didactic of mathematics by focusing on the application rather than on the theory. Mathematical applications dealing with current problems create motives for further study on the theory. A complete example on mathematical analysis of a philosophical structure is presented with beyond expectations results which may be used to provide foundations in education and most human sciences.

INTRODUCTION

Human mind has unlimited capacity in terms of viewing the whole universe as a corn bit or as a complex structure composed of molecules and atoms. A valuable tool of human mind to perform analysis and synthesis of simple or complicated structures is mathematics (James Franklin, 1995 [2]).

Historically mathematics was used for all kinds of structures without exceptions including classical science structures as are the philosophical structures. This is evident for some of the philosophical schools in Athens where there was a sign in the main entrance stating "ΜΗΔΕΙΣ ΑΓΕΩΜΕΤΡΗΤΟΣ ΕΙΣΙΤΩ" which means "the entrance is not allowed to those they do not know geometry". Unfortunately the Roman emperor Iustinianus around 520 A. D., closed the philosophical schools of Athens and this coincides with the time the dark ages begin. Dark ages lasted about one thousand years where people were forced to abolish their ability to search for the truth using scientific methods but instead were guided to accept controversial religious beliefs. Mathematics from renaissance to date, are mainly used to analyze, for example, engineering structures or structures in physics and other scientific fields. However, they are not used much to analyze classic science

structures as are, for example, philosophical structures. The impact of this situation is to maintain a near dark ages era in the field of philosophy and in most human science fields. Consequently, the education system fails to a great extent to convince the student about the usefulness of mathematics as a tool of human mind and also fails to provide the student with the power of this tool. Philosophy on the other hand has to provide the bases to all structures developed by human mind and to do that has to stay itself within human dimensions. From the moment philosophy started dealing with matters as for example, with the absolute truth, and minimized the use of mathematics as a tool, then philosophy was followed a way which is the one we see today and which leads to nowhere. The impact of neutralization of philosophy in our times resulted to the lack of bases for modern education system with unpredictable consequences as is the destruction of the environment.

Based on these thoughts there will be a discussion on the use of mathematics as a tool to analyze structures using simple application examples to improve didactics. Consequently there it will be given two classic science examples using mathematics by presenting two corresponding philosophical structures the one dealing with human spirit and the other dealing with human action.

MATHEMATICS AS A TOOL OF HUMAN MIND

A tool is considered something which facilitates to accomplish a task. Human mind defines the task and starts gathering information and whatever else is needed to proceed with a solution to a problem. If the task, for example is to buy a specific rectangular peace of property then the information to be gathered is the area of the property and the value per square meter. Mathematics will help to analyze the geometrical structure of the property to estimate the area and the economical structure of the task to analyze the money to be paid. The geometrical structure defines two necessary elements of the structure easily obtained by measurements as is the length (a) and the width (b) and the relationship of those two elements to compute the area (A) as follows:

$$A = a.b \qquad (1)$$

The economic structure is also composed of two elements the area as defined and the price per square meter (p) to estimate the property cost (C) which is computed by the relation:

$$C = A.p \qquad (2)$$

$$\text{or} \qquad C = a.b.p \qquad (3)$$

This example could be more complicated if the economical structure would involve a bank loan and it is needed as a task to compute the monthly payments. In all such

cases there is a specific task managed by human mind which brakes it down into specific structures and the mathematics are used as a tool to analyze each structure in specific elements and to determine mathematical relationships among those elements (see Equations 1, 2), or synthesize simple structures into more complicated ones with all necessary elements involved (see Equation 3). Mathematics help to estimate quantities difficult to be obtained directly as is the property cost, for example, from other elements of the structure which are either known quantities, or, are quantities easily measured or estimated. If in Equation 1, the accuracy of computing the area is needed then it must be utilized a more complicated mathematical structure to include a functional part as is Equation 1 and a stochastic part which will describe the distribution of random errors involved on measuring the lengths (a) and (b). This is necessary because quantities (a) and (b) are not true values but they are sampled out from a population of infinite number of possible measurements.

CLASSIC SCIENCE STRUCTURES

Both of structures to be analyzed have been carefully chosen and may be considered to have equivalent value for human sciences as Newton's Law in physics. Both of these structures will be analyzed using mathematics.

Human spirit structure is defined by Plato [8]

"… spirit consists of three components or three states that analytically are: logic, desire, and anger.

Plato also defines education as:

"*the therapy of spirit …and as, when the body is sick, it needs medical treatment, when spirit is sick, it needs education*".

Plato integrates this structure by defining healthy spirit as follows:

"*… logic keeps control over and balance between desire and anger*"

To clarify the meanings of control and balance, Plato [8] gives the following example:
Compares spirit with a car pulled up by two horses - a blind horse representing desire and a crazy horse representing anger - and the coachman - being the logic - who keeps control over and maintains a balance between the two horses in order to move the car to the correct way (which is the way of virtue, [4], [5])

This example about healthy human spirit as a mathematical structure can be expressed with a rectangular triangle where the hypotenuse is logic and the two

other sides represent desire and anger. A mathematical relation of healthy spirit can be established by Pythagoras theorem. This human spirit structure resembles also a three dimensional coordinate system where the three coordinates (X, Y, Z) can be expressed as positions of all points in the three dimensional space. Similarly three components (R, G, B) of primary colors are needed to express any color hue. The same thing could happens with Plato's three components of human spirit where all states of human mind (thought, feelings, joy, happiness, sorrow, imagination, etc.) can be expressed by these three components.

It is important to notice that modern definition of education which is related to the production and transfer of knowledge without any philosophical foundation creates a controversy and thus is maintaining an expanded dark age's era in our times.

Virtue which represents human action as a structure is defined and analyzed in all its extent and in all its depth by Aristotle in his work *The Nikomachean Ethics* [1]. According to Aristotle, Virtue is: *Mesotita* (a mid-way) and is to be found in midway, in between two extreme actions or "badness". Aristotle then gives the following example in order to clarify the structure of virtue:

If bravery is a virtue then the brave person is to be found in midway, between the provocative and the coward person, …*and when one is brave, then the coward will call him provocative because he is beyond coward's capacity, while the provocative will call him coward because he is beneath provocative's capacity…*

Accordingly, one could characterize thrifty as a virtue that is to be found in midway between stinginess and overspending and the stingy will call the thrift as overspender, the overspender will call the thrift as stingy.

Aristotle also defines the person of virtue *as the one who is trying to be a person of virtue* which means that virtue is the effort to maintain actions within the mid-way and which allows extreme actions under certain conditions as is for example, self defence. The important idea about this structure is that it is completely fitted within human dimensions.

MATHEMATICAL ANALYSIS OF MIDWAY OF VIRTUE

Examining more carefully the example that Aristotle gives about the brave person, who is a person of virtue, then he/she will be considered by the coward as provocative, which means that the coward, believing that he/she is a person of virtue, underestimates virtue and therefore mathematically this can be described as committing an error with a negative sign. On the contrary, the provocative considers the person of virtue as coward and accordingly overestimates virtue hence he/she mathematically can be described as committing an error with positive

sign. It is evident that humans, by their own nature make errors because human brain is anatomically based on neurons and such systems are not absolutely correct.

For example, if one walks over a flat road and meet a small obstacle like a rock having the size of a football, then the way one rises the foot to pass over the obstacle is different each time one walks over. There is an optimum way to pass over the obstacle with minimum energy (zero error) which can be approximated with practice but it will never be followed exactly. There is a lower limit and an upper limit to rise the foot to pass the obstacle with optimum energy (correct, virtue) without having a false step. Finally there are many outside these limits cases where it takes place a false step (negative and positive error). How bad the false step is depends on the damage caused to this person (absolute value of error).

It must be nodded that, repeating an effort, neurons are trained and constantly improve their performance but they never become perfect. This example may help to define precisely the borders of wrong and right where wrong occurs by a false step and right occurs with optimum energy.

The Aristotelian midway of virtue has a universal validity, for example, taking into consideration the orbit of the earth around the sun, one may observe that the earth will never follow exactly the same path and there is a midway where orbits of the earth must occur in order to have equilibrium. If the earth gets off such bounds towards the inside, then the earth may collide with the sun, if the earth gets off such bounds towards the outside, then the earth may get lost in space. This example defines also precisely the boundaries of wrong and right where wrong occurs when the earth tends to collide with the sun or tends to get lost in space while right occurs within the midway of orbits which follows until now.

Having this analysis in mind, then human error structure can be expressed mathematically as follows [4], [5]: We may establish an axis X (See Figure 1), consisting of three straight line segments:

(a) The segment on the left is named "Error on the left (M_L)", measures the degree of error or badness of a human being and shows the amount of underestimation of virtue (negative error);
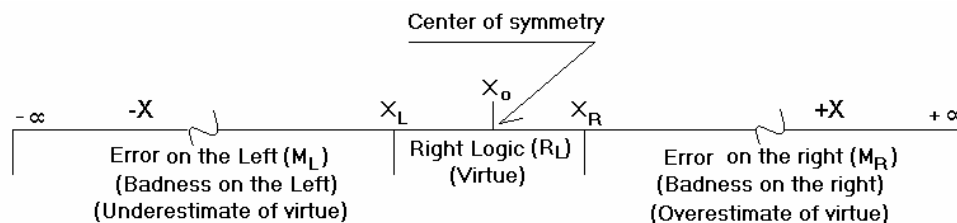

Figure 1. Structural elements ordered along X axis [4], [5].

(b) The intermediate segment in the middle called "Right Logic (R_L)" or virtue (correct).
(c) The segment on the right is named "Error on the Right (M_R)", measures the degree of error or badness of a human being and shows the amount of overestimation of virtue (positive error).
(d) The location X_o with zero error we call *Supreme Being* location because no human being is considered as having zero error.

As shown in Figure 1, this distribution of human error is symmetric with respect to the central point X_o of zero error in the intermediate segment of virtue. However, human error is expanded from point X_L to minus infinity and from point X_R to plus infinity. If humans would be able to estimate precisely their error they would choose a specific location along the X-axis. Notice that many times people estimate such a location as are the political parties, community clubs and their followers. It must also be noted that the location of boundaries of midway of virtue requires a wider consensus (stochastic model) because each individual may have a different opinion on the subject and such a consensus may be valid only if there is a minimum bias in expressing such opinion. Minimum bias may be obtained if the voters have an education focused on developing a healthy spirit as defined by Plato [8]. Such matters about the influence of bias are discussed in detail by [4], [5]. Consensus means democratic procedures and voting which are philosophically founded as an effort to define the mid way of virtue.

It is important to understand that since any human action includes an error (X) it means that this action to a certain degree is correct (Y) so that wrong (X) and right (Y) coexist within such action. If wrong and right are to be quantized then they must be inverse proportional to each other. A simple function to express this is as follows:

$$\mathbf{Y = 1/X} \tag{4}$$

Where (X) is the wrong or error and (Y) is the correct or right.
From Equation (4) it is evident that:

$$\text{for } |\mathbf{X}| \rightarrow \mathbf{0} \quad \text{then} \quad \mathbf{Y} \rightarrow \textbf{infinity.} \tag{5}$$

Since X_L, X_R are located on the common boundary between wrong and right, then both variables X and Y must have exactly the same value ([6], pp. 328) on this common boundary. However, we look for a value in X-axis where:

For boundary X_R : $\qquad \mathbf{X = Y}$ $\hfill$ (6)
and for boundary X_L: $\qquad \mathbf{-X = -Y}$

Then, from Equation (4): $\quad \mathbf{X = 1/X}$ or, $\mathbf{X^2 = 1}$ $\quad$ or, $\quad \mathbf{X = \pm 1}$ $\hfill$ (7)

This determines precisely the borders of the midway of virtue [4], [5] as having values:

$$\mathbf{X_L = -1,} \text{ and } \mathbf{X_R = +1} \tag{8}$$

Considering that such borders are defined by voting and the probability density function of human error is f(x) then as a stochastic model seems to fit to the *Gaussian standard normal distribution* [4], [5].

SUPREME BEING

As indicated by Relation (5), Supreme Being has a virtue with magnitude approaching at infinity and, consequently, it is not possible for this Being to have even minimal badness at all times. Consequently, if we accept that Supreme Being has any of human weaknesses at any time, we immediately depart from the location X=0 and thus we have not just one but numerous such beings like human beings. Putting on the same diagram shown in Figure 2 both the error function X and the correct (virtue) function Y = 1/X (Y - axis is perpendicular to X - axis) one may notice that if X takes values from -1 towards zero, then Y moves towards minus infinity. On the other hand, if X takes values from +1 towards zero, then Y moves towards plus infinity. This indicates that Supreme Being is found in one single location of the X axis and has a virtue which covers all values in universe from minus infinity to plus infinity. This is one and unique location because if departing even with a small amount from location zero, say 0 + e, or, 0 – e, where e is a very
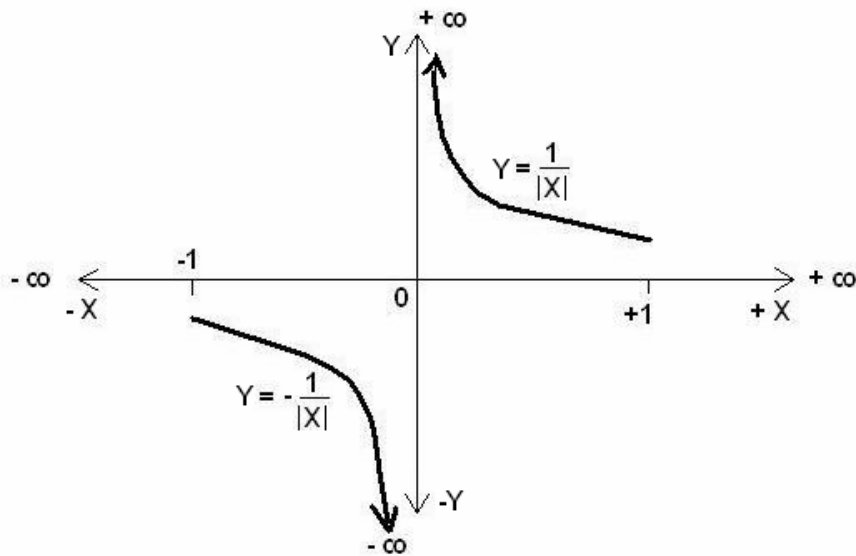

Figure 2. Virtue for the Supreme Being ranges from minus infinity to plus infinity.

small number, then there are many beings in such location with human weaknesses and not just one and unique being.

This analysis may help scientists in classics and theology to make a better interpretation of scripts and thus eliminate human weaknesses and human errors from the Supreme Being. This is also an interesting indication as to where the absolute truth is located.

DIDACTICS

The previous examples were given to help understand that mathematics is a valuable tool of human mind to perform analysis and synthesis of simple or complicated structures. This is very important for those they want to improve didactics. Looking at student's status one may see that about a 10-20% are talented students in mathematics and they understand and learn the theories right away and therefore for those students there is no problem. The majority of students 80-90% need more help to understand mathematics. A didactic method which could help such students is to use application examples from every day's life. A typical such example is the first one given here about estimating the value of a rectangular piece of property. This is known as the R.E. Gross [3] problem solving method. A complete example of this method is given by Manolas E., 2006 [7]. R.E. Gross method if combined with computer programming to obtain quick results on complicated mathematical problems could improve the didactics and could also help pupils from the elementary school to advance (in being smarter than the machine) into the new technology era.

Working on students at a University level one may be the recipient of all problems students have from elementary school to the high school and lykeio (senior high school) in mathematics. Trying to decode the problems students have in this area it is evident the wrong didactics followed through this course. Perhaps the problem can be located at the following practice by most educators in mathematics:
1. They do not realize that mathematics is the science of structures [2].
2. Because of attitude #1, they do not orient their didactics towards applications to create motives to the students but instead they spend their time on theories which for the talented students there is no problem but for most of the students it does not make sense and they miss the substance creating gaps which generate opposite feelings and hateness about mathematics.
3. Many mathematicians particularly the good ones do not have experience about applications of mathematics or they do not like applications.
4. Computer programming in a simple computer language, Visual basic for example, is not present in curricula of elementary and secondary education schools.

If those issues are taken seriously into consideration then it is anticipated to have improvement to the current situation. An ideal improvement could be to obtain a percentage of 60-70% of students to understand and learn mathematics. It must be noted that probability between $\sigma = -1$ and $\sigma = +1$ in standard normal distribution is within this percentage range.

CONCLUSIONS

This work concluded that human mind has unlimited capacity in viewing the universe and a valuable tool to perform analysis and synthesis of simple or complicated structures is mathematics.

A very important issue about mathematics is the expulsion of mathematics out of the classic science studies. The two examples given about human spirit and human action indicate that studying such structures without using mathematics almost 95% of the substance is missing with a result to maintain a dark ages era in most fields of human sciences.

The study of classic science structures using mathematics can help to provide philosophical bases, as for example to education and democratic procedures, and define precisely the boundaries between wrong and right.

Didactics of mathematics is a major problem which can be significantly improved using application case examples based on every day's practice and following an evolved E.R. Gross problem solving model with the addition of computer programming.

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