

Using Spatial Microsimulation for the analysis of social and spatial inequalities

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Abstract

Understanding local impacts of national scale policy requires a clear and coherent knowledge about the effects of the policy in the smallest possible scale. This is the individual or household effects from this policy. Microsimulation is a methodological approach which by simulating in individual level can produce a robust representation of a population under the effects of different policies or events. This understanding is valuable for policy makers, in order to understand the dynamics of a population as well as producing clear “what-if” scenarios. This paper will examine the methodological details of dynamic spatial microsimulation software with modern open-source and object-oriented software engineering structures. This very model is then used for a case study of exploring social and spatial inequalities in education. By using census and panel data (UK Census and BHPS) and incorporating meta-heuristic algorithms, micro-scale datasets are produced and then used for custom projections. Furthermore, this work will discuss and compare different algorithms that can be used as well as illustrate benchmarks of their potential use. Finally, topics of further development will be discussed along with potential use of such models in policy making and research. Simulating geographical space has always been difficult, but if we incorporate human actions and reactions, becomes a huge challenge due to its complexity and chaotic nature.

1. Introduction

The aim of this research is to illustrate the use and applications of spatial microsimulation. The objectives are the discussion of spatial inequalities for education, the illustration of the need for a spatial microsimulation model, and the development of a well defined and robust model for preparing data at the smallest possible geographical area for a better research and policy evaluation.

In the first part, we will discuss higher education loans as a policy example for the manipulation of social and spatial inequalities in access to education. Next will discuss the method of spatial microsimulation, the data availability and form and the MSM 1.0 model will be illustrated and discussed. The next and final part will be the discussion of the outcome of the model along with a simple spatial analysis on the geographical area of interest for better understanding.

2. Higher education Loans

It is well known that education is one of the major factors for reducing inequalities in the social layer. The importance of education as a whole is outside of the scope of this research. The main objective of this research is the understanding and illustration of inequalities in educational finance schemes and more specific the higher education loans for full time students. There is available funding for higher education students in a form of education loans which have to be repaid after graduation. The available schemes provide financial support for the costs of studying and the student won't have to pay back anything until he/she has a constant salary. The available financial support for full time UK students comes in forms of “Student Loans” (SL from now on) and grants from the government as well as bursaries from universities and colleges (DirectGov 2008a). Students receiving SL from government won't have to start paying it back until their annual salary from a job reaches £15,000. Those SL come in two types:

- Loans to cover the tuition fees (around £3,000)
- Loans to help towards accommodation and living costs (around £6,300)

According to government student funding authorities (DirectGov 2008b) the total amount of money granted to a student is based on household income. Students living with their parents or having a spouse/partner, may be asked to be supported by the household as they partly fulfil the criteria for self financing. The main body of the income assessment is the Local Education authority. This authority will count the student's income which includes non-earned income like interests from savings but not casual or part-time earnings during the period of the course. Also LEA will count income from the parents or partner as this is a measure to understand and evaluate the financial background of each student. This will be counted if the student is a “dependent” or “independent” member. Parents' household income will be assessed in order for an

educational loan to be granted for the student. The assessed parental contribution is the contribution of parents towards student's expenses and is deducted from the Maintenance Loan once household income is in excess of £60,006. Table 1 depicts the overall connection between household income of a student along with the expected parental contribution towards education and the associated maintenance grant and loan for living expenses.

Household Income	Parental Contribution	Maintenance Grant	Maintenance Loan	Total
17,000	0	3,265	2,810	6,075
25,000	0	2,835	3,365	6,200
30,000	0	2,002	3,365	5,367
34,450	0	1,260	3,365	4,625
40,000	0	998	3,627	4,625
50,000	0	524	4,101	4,625
60,005	0	50	4,575	4,625
61,061	0	0	4,625	4,625
65,000	414	0	4,211	4,211
70,000	940	0	3,685	3,685
72,034	1,155	0	3,470	3,470
80,000	1,155	0	3,470	3,470

Table 1: Household income categories for educational loans

With a simple graphical representation of the table above it is easily concluded that as the household income increases, the available money to the potential student decreases, indicating that household is responsible from a point onwards for student expenses. Those indicate the expected parental contribution towards the education of a student, which generally is much higher and includes more financial support from the household towards the living expenses of the student.

The scheme of student loans, offers a financial support to the student for education and living costs for the period of the selected course. This widespread accessibility to financial support raises some concerns about ability of repayment and student dept.

The office of national statistics (Statistics 2004) produced a report about social inequalities in 2004 and clearly illustrates the big picture of financial debt of people aged 15 to 24. According to this work, this age group has a 0.45 mean debt to income ratio, which is the highest ration between all 6 age groups. In the academic year of 2002 – 2003 according to (Callender, Wilkinson et al. 2003) nearly half the total income of undergraduate students aged under 25 came from student loans. By the end of the academic year, the 87 per cent were in debt and the average debt over all students was £5500. Students from lower social classes borrowed more than the average amount of commercial credit and were least likely to have any savings offset against their debts or to get any financial help from their families. Data from Student Loans Companies indicate that at the end of financial year 2002 – 2003, there were 2.6 million people with student Loans of whom 10 per cent owed two or more months of repayment (DfES 2003). This was the overall picture of debt aspect of student financial support towards higher education. It is more than clear that some steps have to be made towards better understanding and association between income and availability of loans.

Apart from the student loans availability and size, there is also other issue with the educational financial support scheme. This is the issue with inequality to access to higher education by young people (Hefce 2005). This report measures participation of young people to higher education according to where they live. One of the first findings is that when researching participation in education, using areas of the size of wards appears to work well in reliably capturing participation neighbourhoods, as it's rare for wards to be internally mixed in terms of young participation. The key finding of this research is the deep divisions in the participation of young people to higher education, changes by where they live. This is a clear indication that participation is highly correlated with geographical characteristics of individuals and any effort to understand the problem should include geographical analysis.

This is strong evidence that further examination of participation to higher education has to be made and that new techniques and methods have to be implemented in order to offer a diversity of approaches for understanding and evaluating a problem with spatial characteristics. One of those new methods with potentials and well recognized strengths of analysing spatial data is spatial microsimulation.

3. Method

3.1 Spatial Microsimulation

Spatial microsimulation is a relatively novel approach in simulation and incorporates the notion of space in a scientific simulation. A simple and quick definition of Scientific Simulation could be: "*An abstract simplified representation of reality for scientific reasons and study*". It is a simplified version of real world in

order to perform analysis and interpretation. Because it would be impossible to transform nature and reality to absolute numbers in order to study it, we use simplified versions of it, which have the same rules and objects. As Troitzsch describes it (Troitzsch 2006), scientific simulation, in social sciences, is a 3rd symbol system beside natural language and mathematic formalization.

Microsimulation has first been formulated by Orcutt and his colleagues (Orcutt, Merz et al. 1986) who had already from 1957 (Orcutt 1957) introduced the general idea of simulating micro-units. Microsimulation as an idea is a technique for building large-scale datasets by simulating at the microscale (individuals, households or business). Microsimulation in economics is a well established method with wide range of applications such as: tax/benefit, budget analysis, measurement of poverty and policy impact assessment (Ballas, Clarke et al. 2005). A microsimulation model simulates individuals at the micro level and uses its attributes to determine the impacts of a policy. Ballas and Clarke (2003) point out in a paper reviewing microsimulation in regional science, the first geographical application of microsimulation was developed by Hägerstrand (1967) who employed micro-analytical techniques for the study of spatial diffusion of innovation. Further, the basis for spatial microsimulation of households and individuals was founded in the 1970s, when Wilson and Pownall (1976) addressed the aggregation difficulties that were associated with traditional comprehensive spatial models of urban systems. Since then there was comparatively little new work in the area. Amongst the notable exceptions has been the work of Birkin & Clarke (Birkin and Clarke 1988; Birkin and Clarke 1989) who developed the first model of this kind in Britain building on Wilson & Pownall's theoretical foundation by actually developing and applying a geographical microsimulation model. Since then there has been an increasing trend in the development and use of spatial microsimulation by academics and policy makers like SimBritain (Ballas, Clarke et al. 2005), and Micro-MaPPAS (Ballas, Kingston et al. 2006).

In this research we construct a spatial microsimulation model for the analysis of educational inequalities in Yorkshire and Humber.

This research constructs a new spatial microsimulation model which will take under consideration socio-economic characteristics of population and correlate them with educational related data in order to understand and analyse. The produced output would be a micro-dataset which is a clean and robust description of a geographical area, in the micro scale (smallest possible geographical area). For confidentiality reasons, data in small geographical scales do not exist. With the use of spatial microsimulation, we are able to construct these types of data, in order to perform analysis. Availability and type of data is crucial for the construction of microsimulation. Before describing the model and its characteristics, it is necessary to understand the datasets used along with their characteristics and format.

3.2 Data sources

The research uses data from the UK Census of Population (2001) (Census.of.Population.Programme 2006) as well as panel data from the British Panel Household Survey (BHPS). The census data will form small area constraints of the spatial microsimulation model and the panel data will add information to the virtual individuals that will be created. Constraints will confirm the basic numbers of the area such as: the amount of educated people, the amount of households and total number of individuals. The panel data will add information such as what is the income of an individual or what are his educational qualifications as well as probabilities on the likely future attributes of each individual. The Census of Population has been chosen as the most reliable information as far as it concerns general purpose information about a population in the UK. According to the Quality Profile document of BHPS (IISER 2006) the central purpose of the British Household Panel Survey (BHPS) is to understand the dynamics of change experienced by the population of Great Britain and its evolution over the lifetime of the survey, from its start in 1991. In contrast to most cohort studies, their samples cover the whole population, and not simply narrow age ranges. While BHPS aims to provide cross-sectional population estimates for the lifetime of the study, its central aim is to facilitate longitudinal research. For related reasons it is primarily produced in order to make micro-data sets available to a wide range of secondary analysts across a range of social science disciplines, and for policy research.

The UK Census of population provides the most authoritative social accounting of people and housing in Britain and is a unique source of data for the social sciences (Dale 1993). The Census records demographic and socio-economic information at a single point in time and is normally carried out every ten years. Census datasets describe the state of the whole national population and are extremely relevant for the analysis of a wide range of socio-economic issues and related policies. The topics covered by the Census are determined, amongst other factors, by the necessity to preserve comparability over time and the need for timeliness (Dale 1993). Census statistics in the UK are collected by the Office for National Statistics

(Office.for.National.Statistics 2006). The national Census of the UK enumerates the population every ten (10) years except from 1941 due to Second World War.

The census data are available via the Census Area Statistics Website (Census.of.Population.Programme 2006). The main dataset used for this study from the Census of Population. The model evaluated all available variables and processed all areas in the smallest possible detail which is the Output Area – a reporting unit of approximately 120 households on average. For map representation, a greater scale (wards) was selected in order to preserve a more coherent illustration and understandable maps. Smaller areas in dataset had some data blurring embedded from the Census of Population, in order to preserve confidentiality as the likelihood of private data disclosure is greater as detail increases (Output Areas). This has an impact on the results of the model as there is no perfect matching up to every individual in every output area.

Apart from numerical datasets, this research uses digital map files for displaying the outputs of the model. All data have been downloaded from “UK Borders” (Edina 2008). The type of analysis and the required methodology, needs some well formatted and error free digital boundaries in order to perform spatial analysis and illustrate the outputs. One of the innovative parts of this research is the incorporation of a smart “boundary effect” elimination mechanism. Because we research educational institutions (points) along with their effect with Wards (polygons), a way to represent the real effects of those transactions, was needed. This is the reason; a methodological framework for checking outside the boundary of a geographical area was developed. Here is an example to illustrate and explain the followed procedure and the logic behind it.

Error! Reference source not found. depicts the area of interest (Yorkshire and Humber) along with the additional post code districts which have been added in order to evaluate the possible existence of academic institutions in those areas. The Academic institutions have been digitized according to their post-code. So as **Error! Reference source not found.** shows, if for example we have an institution (1) outside the area of interest, but still this influences (has a catchments area which overlaps with part of area of interest), then this post code district is also added initially in the datasets, just for the creation of the institution, and then the post code polygon will be removed. This eliminates the boundary effect which would had the household (2) which wouldn't be close to an academic institution in our model, but this is not the case in reality as can be seen in **Error! Reference source not found.**.

Those were the tabular and geographical datasets used in this research. Those data were used in combination with a novel combinational optimization technique, which connects and reprocess them for obtaining an accurate representation of localities (output areas) for further research. This process is called simulated annealing and is described in the following part after a brief description of the object orientation approach used to represent households and individuals in the model.

3.3 Object orientation and geographic microsimulation

Information systems development has progressed through a number of different development paradigms as our understanding of how best to model the world has improved. there has been a natural progress from the procedural paradigm, to the modular paradigm, to the data abstraction paradigm, and presently to the object orientation paradigm for development (Stroustrup 1997). Object orientation is one of the most natural ways to understand and reconstruct the real world and its interactions. The main idea of Object Orientation (OO from now on) is viewing and modelling the world as a set of interacting and interrelated objects. Objects are physical or conceptual entities with an **identity**, **properties**, and **operations**, and can be created and destroyed.

- Operations define the behaviours an object can perform
- Properties define the state of an object
- Identity are all those characteristics that differentiate one object from an other,

By using OO methodologies and programming languages, we benefit from its advantages. Those include: **inheritance**, **encapsulation**, **abstraction** and **polymorphism**. It is not necessary to analyse those well known techniques, but we will emphasize on the representation of educational reality as objects with methods and attributes. Individuals and geographical entities will form objects which will change over time. Those objects will have attributes which will characterize them, and methods which will affect them.

By using object orientation, the system will be easily updatable, in case of expansion to include less or more units. It will be easily adaptable to new geographies if there is a need for implementation on a new area. It will be easily customizable by any other social scientist who wishes to use the code and/or the model to perform similar fitting processes.

3.4 Simulated Annealing

Until now for simple combinational and optimization processes Hill climbing algorithm was the first choice. This algorithm though, in large scale datasets and complex search space problems had some disadvantages, and when for example the aim was to find the global minimum, it was trapped in local minimum or a

“plateau” of the available search space. In order to overcome the problems of simple hill climbing, a more complex algorithm was used. Its name is Simulated Annealing. Simulated Annealing (SA) takes its name from the annealing in solid materials (Metropolis, Rosenbluth et al. 1953).

If you heat a solid past melting point and then cool it, the structural properties of this solid depend on the rate of cooling. If the liquid is cooled slowly, large crystals will be formed. Metropolis’s algorithm simulated the material as a system of particles. The algorithm simulates the cooling process by gradually lowering the temperature of the system until it converges to a steady, *frozen* state.

Kirkpatrick et al (Kirkpatrick, Gelatt et al. 1983) took the idea of the Metropolis algorithm and applied it to optimisation problems. The idea was to use simulated annealing to search for good solutions and advance towards an optimal solution. Simulated annealing is described in many books. An easy description of the algorithm can be found in (Dowsland 1995) which contains introductory material and examples of Simulated annealing.

Simulated annealing is more robust than hill climbing because it allows some uphill steps that can foster movement away from local minima (or maxima).

In contrast with hill climbing, simulated annealing chooses a random move from the neighbourhood (hill climbing chooses the best move from all those available). If the move is better than its current position then simulated annealing will always take it. If the move is worse then it will be accepted based on some probability. According to the law of thermodynamics, at temperature t the probability of an increase in energy of magnitude, δE , is:

$$P(\delta E) = \exp(-\delta E / kt)$$

Equation 1: Law of thermodynamics

(k is a constant known as *Boltzmann’s* constant). If the energy has decreased, then the system moves to this state. If the energy has increased then the new state is accepted using the probability returned by:

$$P(\delta E) = \exp(-\delta E / kt)$$

Equation 1. A specific number of iterations are done at each temperature and then the temperature is decreased. This is repeated until the system freezes into a steady state (temperature=0).

It is common to drop the *Boltzmann* constant because this was only introduced into the equation to handle different materials. Therefore, the probability of accepting a worse state is:

$$P = \exp(-c / t) > r$$

Equation 2: Probability of acceptance

Where c is the change in the evaluation function, t is the current temperature and r is a random number between 0 and 100. As the temperature of the model decreases the probability of accepting a worse move is decreasing. When the temperature is zero then only better moves will be accepted which effectively makes simulated annealing act like hill climbing (*advance only towards better solutions = advance only towards the top of a hill*).

The idea behind simulated annealing is to use conventional hill-climbing techniques, but occasionally take a step in a direction that does not satisfy the goal function (Azencott 1992). This can sometimes avoid local optimum dead-locks and lead to a good solution in the long-term. In other words, when we select a population and make multiple recombination in order to satisfy the objective function of a geographical area, we sometimes select individuals which do not immediately satisfy the conditions, but this sometimes leads the computational algorithm outside of a dead-lock. In simulated annealing, as time passes, the probability that a non-satisfactory record can be accepted is gradually reduced. So as time passes, the algorithm becomes less and less acceptable to unsatisfactory solution.

This algorithm was incorporated in the development of MSM 1.0 model which is used for this research and proved to be a good choice as it produced better results than simple combinational algorithms like Hill climbing. **Error! Reference source not found.** depicts the final resulted error of 2 random areas throughout the iterations of recombining data with Simulated Annealing. As we can see, the algorithm is less and less tolerant as it runs.

This was the algorithm that will be used for the model in order to select individuals from a pool of individuals (BHPS data), and construct a micro dataset according to rules set by Census of population.

3.5 Model

In the context of this research, a model was created with the use of Java programming language (Sun Microsystems 2008). This programming language has been accepted by the academic community as one of the few popular and powerful languages to construct and produce models and simulations. The next part will

illustrate the main java advantages and the reasons it has been chosen as the main programming language for this project.

None of the main powerful programming languages is simple to learn and implement. Although java has simple rules, classes and rules in order to focus in the implementation and the logic of the code rather than the understanding of the language. Object oriented languages like C++ and java involve complexity in structures and methods but java handles this complexity in a neat and “clean” way.

Scientific models like the one we create handle multiple datasets and actions in a distributed and parallel way. This demands a language that will handle multiple events and actions in a simple and coherent way. Java offers a number of ways to handle those objects and coordinate their inputs and outputs easily and secure with accurate metrics and co-operative structures.

Portability and Operating System independence is also one of the advantages of Java programming language. The famous quote “*Write once, run everywhere*” is common among java developers. Because of the notion of Virtual Machine (Lindholm and Yellin 1999) the java code is portable and can be compiled and run in any machine. The machine that transforms the code to machine language is portable and so it is not dependent on the machine of the user. The same code will run on different machines producing the same results and interacting with user in the same way.

Furthermore, java is famous for the multimedia handling libraries it includes. Images sound video and graphics can interact with the user providing a rich client experience. This is crucial if representing and demonstrating complex notions like microsimulation models.

Finally, java offers the ability to use the network, internet and mobile technology in a very efficient and secure way. Java applets are java Mid-Lets are structures that can easily attached on web pages and mobile phones accordingly. This offers an advantage of expandability and connectivity between multiple data and clients. This list of advantages could include many more parts, but for the scope of this research we will not expand the discussion further.

3.6 Analysis of output

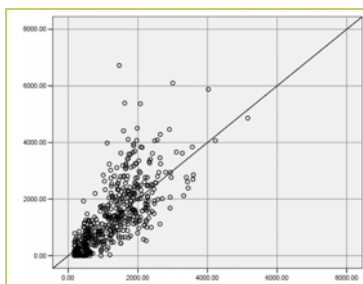
The model was developed in Java and run on a Sun Grid Networked computer. The highly computational power which was available, enabled the model to run in parallel and recombine many areas at the same time. The following graphs (Graph 2, Graph 3, Graph 1) depict the error of the model after 1000, 2000, and 4000 recombination for every area. The error was calculated by plotting in the same graph the actual (census) and the modelled (MSM1.0) values of every area for every variable and then calculating the algebraically deviation from the actual values. As it can be easily concluded, the more the iterations, the less the error of the model. For iterating more, we needed a highly powerful computer that could handle the recombination processes. This was done by using the Grid Computing facility of University of Sheffield.

Map 1 depicts the produced results after 4000 iterations of the MSM 1.0. This map illustrates the mean annual household income for every household in every ward of Yorkshire and Humber. The microsimulation method used Output Areas for simulating every household, but for illustration purposes we used Ward geographical boundaries.

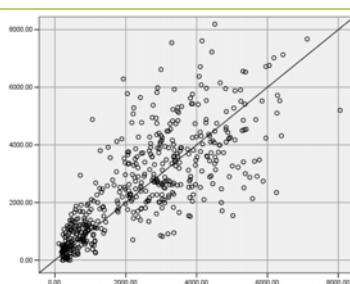
The mean annual household income for a household in Yorkshire and Humber is £26440.

Map 2 shows the amount of households above the mean annual household income and Map 3 the amount of households below this threshold. Both maps have an additional layer of the higher academic institutions of Yorkshire and Humber for analysis reasons.

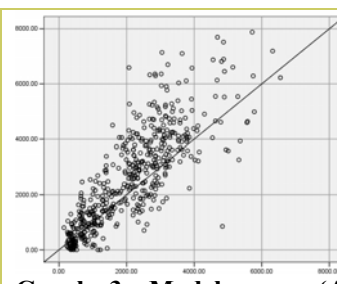
Map 5 and Map 4 illustrate the amount of households with individuals between 16-17 and 18-19 age respectively. This is for evaluating how many households have potential students which will be potentially needed government funding in the form of higher education loans. Graph 4 illustrates the distribution of the amount of household income in Yorkshire and Humber along with the educational loans categories. As we can see, the lower income households are almost all under the first and the second category of loans. For implementing a more fair and equally distributed policy, those households could also be categorized in more than two categories. This would require a re-evaluation of government policy for eligible households for educational loans in the smaller income categories.



Graph 1: Model error (Actual versus modelled) after 4000 iterations



Graph 2: Model error (Actual versus modelled) after 1000 iterations



Graph 3: Model error (Actual versus modelled) after 2000 iterations

4. Using MSM 1.0 for policy implications

Spatial microsimulation is a very useful tool for analysing the effects of national scale policies in the smallest possible scale. In this research we analysed the results of higher education loan eligibility of households from Yorkshire and Humber by simulating the population in Output area level, and then analysing the effects of this policy. Apart from simple policy analysis, a further method can be used to estimate the after effects of a possible change in the national policy. This is, if the policy changes, what would be the effects in the micro-scale. If for example the eligibility scales have changes, how many households with potential students would be eligible for Higher education loans.

This is a very useful methodological tool which may be used in policy evaluation process, if a more detailed analysis of the effects is needed. Powerful enough, this method may result in a more equally distributed policy towards educational loans by re-adjusting the scales and the limits for every loan category. This will provide a more equal distribution of government resources towards households and potential students, leading to a uniformed participation from all social classes to higher education. This will eventually lead to potential graduates from all social classes, contributing to economy in the area.

5. Concluding comments

This research is currently a work in progress and its aim is the illustration of the method and the model under a policy evaluation scenario. By evaluating the use and the development of a spatial microsimulation model like MSM 1.0, we contribute to the effort of developing a robust and automatic way of evaluating policy implications, and analysis with geographical respect in the smallest possible scale. This may appear to be a valuable tool in the arsenal of policy makers as understanding the implications may lead to a re-evaluation of a policy for a better calibration of income levels.

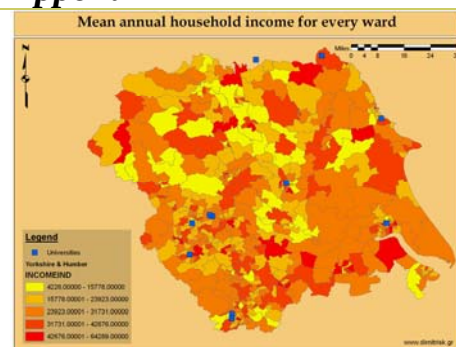
The spatial microsimulation model as a method is available for any scientist to experiment and understand the potential use it may have for advanced combinational optimization processes. It may easily adapted and alerted to fit the requirements of any scientist with tabular formatted data. The source code would be available, and any alterations under GPL v3.0 (Foundation 2008) are welcomed.

Acknowledgements

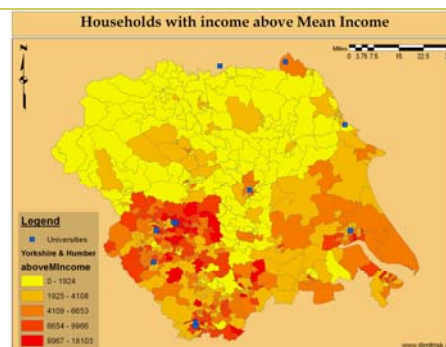
The Census data are provided through the Census Dissemination Unit of the University of Manchester, with the support of the ESRC/JISC/DENI 2001 Census of Population Programme.

All digitised boundary data are Crown and ED-LINE copyright.

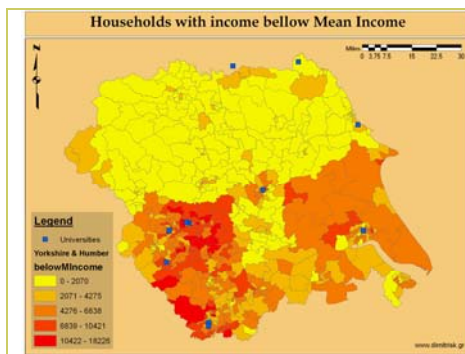
Appendix



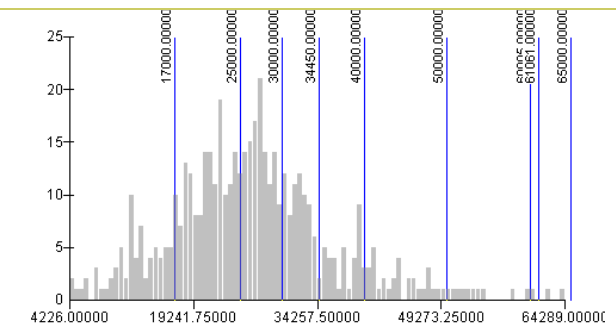
Map 1: Mean annual household income for every ward



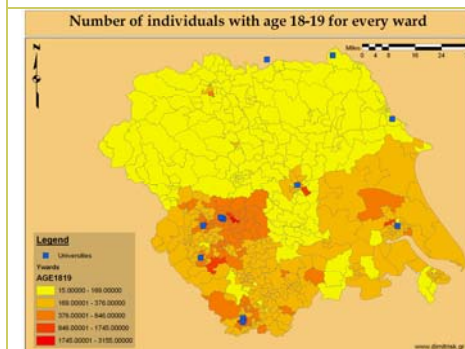
Map 2: Households with annual income above Mean Income level



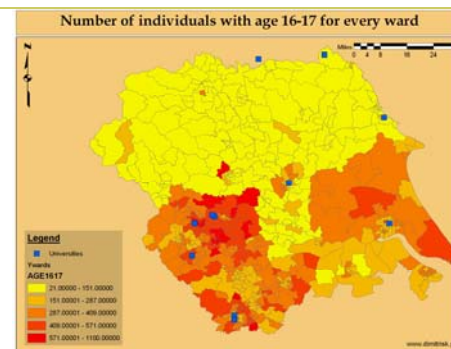
Map 3: Households with annual income below Mean Income level



Graph 4: Mean household income distribution of wards in Yorkshire & Humber



Map 4: Individuals from age group 18 - 19 for every ward in Yorkshire and Humber



Map 5: Individuals from age group 16 - 17 for every ward in Yorkshire and Humber

Classification Statistics	
Count:	496
Minimum:	4226.00000
Maximum:	64289.00000
Sum:	13114130.00000
Mean:	26439.77823
Median:	25900.50000
Standard Deviation:	9874.87000

Table 2: Classification Statistics of wards in Yorkshire & Humber

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